

Evolving to type Ia supernovae with long delay time

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ABSTRACT

Recent investigations on the delay time of type Ia supernovae have set useful constraints on the progenitors of type Ia supernovae. Here we have calculated the evolution of close binaries consisting of a white dwarf and a main-sequence or subgiant companion. We assume that, once Roche lobe overflow occurs a small fraction of the lost mass from the system forms a circumbinary disk, which extracts the orbital angular momentum from the system through tidal torques. Our calculations indicate that the existence of circumbinary disk can enhance the mass transfer rate and cause secular orbital shrinkage. The white dwarf can grow in mass efficiently to trigger type Ia supernovae even with relatively low-mass ($\lesssim 2M_{\odot}$) donor stars. Thus this scenario suggest a new possible evolutionary channel to those type Ia supernovae with long delay time $\sim 1 - 3$ Gyr.

Subject headings: stars: general — binaries: close — stars: mass loss — stars: evolution — circumstellar matter

1. Introduction

Many works have suggested that type Ia supernovae (SNe Ia) can be used as the standard candlelight to determine the cosmological distances, providing the strong evidence that the low red-shift Universe is accelerating (e.g. Riess et al. 1998; Perlmutter et al. 1999; Riess et al. 2004). It is widely believed that SNe Ia are thermonuclear explosions of accreting CO white dwarfs (WDs) when their masses grow beyond a critical mass (Hoyle & Fowler 1960). However, the nature of the progenitors and the related accretion processes have still remained unclear. Several possible evolutionary scenarios for SN Ia explosions have been proposed so far. At present, there exist Chandrasekhar (Ch) mass model (Woosley & Weaver 1986) and sub-Chandrasekhar (sub-Ch) mass model (Nomoto 1982b) for the critical mass

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of the exploding WDs. Furthermore, both double degenerate model (DD) (Iben & Tutukov 1984; Weebink 1984) and single degenerate model (SD) (Nomoto 1982a) have been also proposed as the candidates for progenitors of SNe Ia (for a review see Branch et al. 1995). For the DD model, previous works indicated that the expected accretion rates may cause the accretion-induced collapse (AIC) of the WDs and the formation of neutron stars instead of SN Ia explosions (Nomoto & Iben 1985; Saio & Nomoto 1985; Timmes, Woosley & Taam 1994).

Binary evolution investigations adopting the SD/Ch scenarios obtained two types of progenitors of SNe Ia. One is close binaries with a main-sequence or subgiant companion of mass $\sim 2 - 3.5M_{\odot}$ and an initial orbital period of ~ 1 d (Li & van den Heuvel 1997; Langer et al. 2000; Han & Podsiadlowski 2004). The required accretion rate for steady nuclear burning onto WDs could be obtained with thermal-timescale mass transfer from the more massive donor stars. The other is wide binaries with a red giant of masses $\sim 1M_{\odot}$ and an initial orbital period of ~ 100 d (Hachisu, Kato & Nomoto 1996; Li & van den Heuvel 1997). However, mass accretion in these systems may be unstable due to the thermal-viscous instability in the accretion disks (Cannizzo 1993).

The various progenitor scenarios can be tested by comparison of the expected and measured distribution of the time delay between the formation of the progenitor systems and their explosions. Recent high- z supernova observations suggested the mean delay time of $t \sim 2-4$ Gyr (Strolger et al. 2004, 2005; Gal-Yam & Maoz 2004; see however Barris & Tonry 2006; Förster et al. 2006). However, the main-sequence lifetime of $\gtrsim 2M_{\odot}$ star should be $\lesssim 1$ Gyr in most of the SD/DD SNe Ia progenitor scenarios (Han & Podsiadlowski 2004), and the mass transfer timescale ($\lesssim 100$ Myr) between the components of binary can be neglected. This led Belczynski et al. (2005) to conclude that only SN Ia progenitors in the DD scenario has a characteristic delay time of ~ 3 Gyr. Therefore, if the SD/Ch scenario really works for (at least part of) SNe Ia, a progenitor system with a relatively low-mass donor star is needed.

Supersoft X-ray sources (SSS), originally discovered by the *Einstein* satellite in the Large Magellanic Cloud (Long, Helfand & Grabelsky 1981), may be the observational evidence of the progenitors of SNe Ia (Kahabka & van den Heuvel 1997). A steady-state nuclear burning model on the surface of the accreting WD in a binary was used to interpreted the SSS CAL 83 and CAL 87 by van den Heuvel et al. (1992), who suggested that a near-main-sequence secondary star with a mass of $1.3 - 2.5M_{\odot}$ could provide the required accretion rate for steady nuclear burning via thermal-timescale mass transfer. However, the catalysmic variable SSS J0439.8–6809, J0537.7–7304 and 1E 0035.4–7230 have very small mass ratios and orbital periods ($\sim 3 - 4$ hr) (Spruit & Taam 2001, and references therein), which are

significantly less than that required for a binary with thermal-timescale mass transfer at a rate $\sim 10^{-7} M_{\odot}\text{yr}^{-1}$. van Teeseling & King (1998) proposed that there may exist an efficient mechanism (irradiation-driven mass loss) to enhance the mass transfer rates in these SSS.

The purpose of this paper is to explore the possible progenitor systems for SNe Ia with delay times of a few Gyr in the SD/Ch model. Enlightened by the original works of Spruit & Taam (2001) and Taam & Spruit (2001), we consider the orbital angular momentum loss through the tidal interaction of binary system with a circumbinary (CB) disk during the mass transfer in a close binary. Our previous works indicate that the CB disk is an efficient mechanism extracting angular momentum from the binary system (Chen, Li & Qian 2006; Chen & Li 2006), which may enhance the mass transfer rates and help mass accumulation on the WD. The prescriptions of binary evolution calculations are described in section 2. In section 3, we present the numerically calculated results for the evolutionary sequences of the WD binaries. Finally, we discuss and summarize our results in section 4.

2. Model

We consider a binary system consisting of a CO WD (of mass M_{WD}), and a main-sequence or subgiant companion (of mass M_{d}) with solar chemical composition ($X = 0.7$, $Y = 0.28$, $Z = 0.02$). Mass transfer will occur via Roche lobe overflow of the companion due to nuclear expansion of the star or orbital shrinkage. We have calculated the evolution of WD binaries adopting an updated version of the stellar evolution code developed by Eggleton (1971, 1972) (see also Han et al. 1994; Pols et al. 1995). In the calculations we take the ratio of the mixing length to the pressure scale height to be 2.0. We include the following mass loss processes and orbital angular momentum loss mechanisms during the mass exchange.

2.1. Mass accumulation efficiency

The key factor for the growth of the WD mass is the accumulation ratio α of the accreted hydrogen converted into heavier elements. Unfortunately, there are large uncertainties in estimating the values of α . For the fraction α_{H} of the transferred mass during hydrogen burning, we adopt the prescription by Hachisu et al. (1999) and Han & Podsiadlowski (2004). If the mass transfer rate \dot{M}_{d} is higher than a critical value \dot{M}_{cr} , we assume that hydrogen is converted into helium at a rate limited to \dot{M}_{cr} due to the strong optically thick winds from the WD. The critical mass transfer rate can be written as

$$\dot{M}_{\text{cr}} \simeq 5.3 \times 10^{-7} \left(\frac{1.7 - X}{X} \right) \left(\frac{M_{\text{WD}}}{1M_{\odot}} - 0.4 \right) M_{\odot}\text{yr}^{-1}, \quad (1)$$

where X is the hydrogen mass abundance of the accreted matter. Below this value, all hydrogen is assumed to be burned into helium, until the mass transfer rate becomes less than $\dot{M}_{\text{cr}}/8$, and strong hydrogen shell flashes occur (Kovetz & Prialnik 1994). Thus

$$\alpha_{\text{H}} = \begin{cases} -\dot{M}_{\text{cr}}/\dot{M}_{\text{d}} & , \quad -\dot{M}_{\text{d}} > \dot{M}_{\text{cr}}, \\ 1 & , \quad \dot{M}_{\text{cr}} \geq -\dot{M}_{\text{d}} \geq \dot{M}_{\text{cr}}/8, \\ 0 & , \quad -\dot{M}_{\text{d}} < \dot{M}_{\text{cr}}/8. \end{cases} \quad (2)$$

After the gradual increase of the mass in the helium layer on the surface of the WD, helium ignition occurs. A part of the envelope mass is expected to be blown off due to the helium-shell flashes (Kato, Saio & Hachisu 1989). For the helium mass accumulation ratio α_{He} , we adopt the prescription given by Hachisu et al. (1999),

$$\alpha_{\text{He}} = \begin{cases} -0.175 (\log \dot{M}_{\text{He}} + 5.35)^2 + 1.05, & -7.3 < \log \dot{M}_{\text{He}} < -5.9, \\ 1 & , \quad -5.9 \leq \log \dot{M}_{\text{He}} \leq -5. \end{cases} \quad (3)$$

Summarize the above prescriptions, the mass growth rate of the CO WD is $\dot{M}_{\text{WD}} = -\alpha_{\text{H}}\alpha_{\text{He}}\dot{M}_{\text{d}}$. The mass lost rate from the binary system can then be written as $\dot{M} = (1 - \alpha_{\text{H}}\alpha_{\text{He}})\dot{M}_{\text{d}}$.

2.2. Orbital angular momentum losses

During the mass exchange in WD binaries some fraction of the transferred matter from the donor star may be lost from the system in various ways. For example, high velocity outflows were observed from the ultraviolet spectra of RX J0513.9-6951 and CAL 83 (Gänsicke et al. 1998). Part of the lost matter may form a disk structure surrounding the binary system rather leave the binary system (van den Heuvel & de Loore 1973; van den Heuvel 1994). The optical and UV spectrum of the SSS RX J0019.8+2156 indicate the presence of circumbinary material (Kuduz et al. 2002; Hutchings et al. 2001). Here we assume that a small fraction δ of the mass lost feeds into the CB disk surrounding the binary system at its inner radius r_{i} . Tidal torques are then exerted on the CB disk via gravitational interaction, extracting orbital angular momentum from the binary system (Spruit & Taam 2001; Taam & Spruit 2001). The angular momentum loss rate via the CB disk is (Chen & Li 2006)

$$\dot{J}_{\text{CB}} = \gamma \left(\frac{2\pi a^2}{P_{\text{orb}}} \right) \delta \dot{M} \left(\frac{t}{t_{\text{vi}}} \right)^{1/3}, \quad (4)$$

where $\gamma^2 = r_{\text{i}}/a$, t is the mass transfer time, P_{orb} and a are the orbital period and the separation of the binary, respectively. In the standard " α viscosity" prescription (Shakura & Sunyaev

1973), the viscous timescale t_{vi} at the inner edge in the CB disk is given by $t_{\text{vi}} = \frac{2\gamma^3 P_{\text{orb}}}{3\pi\alpha_{\text{SS}}\beta^2}$, where $\beta = H_{\text{i}}/r_{\text{i}}$, α_{SS} and H_{i} are the viscosity parameter and the scale height of the CB disk, respectively.

In addition, we assume that the other part $(1 - \delta)$ of the lost mass \dot{M} is ejected in the vicinity of the WD in the form of isotropic winds or outflows, carrying away the specific orbital angular momentum of the WD. The mass loss in the donor’s wind and its effect on the change of orbital angular momentum are comparatively negligible.

3. Numerical Results

We incorporate the prescriptions in last section into the stellar evolution code, and calculate the evolution of WD binaries with initial parameters of the WD mass $M_{\text{WD,i}}$, donor mass $M_{\text{d,i}}$, and orbital period $P_{\text{orb,i}}$. Once the WD mass M_{WD} grows to $1.4M_{\odot}$, we stop the calculation and assume a SNe Ia occurred. In our calculations, we set $\gamma = 1.3$, $\alpha_{\text{SS}} = 0.01$, and $\beta = 0.03$ (Chen & Li 2006).

Examples of the mass transfer sequences are shown in Figs. 1 and 2. We plot the calculated evolutionary sequences for a system with $M_{\text{WD,i}} = 0.8M_{\odot}$, $M_{\text{d,i}} = 1.5M_{\odot}$, and $P_{\text{orb,i}} = 1$ d in Fig. 1. The solid and dashed curves correspond to the cases of $\delta = 0.01$ and 0, respectively. If no CB disk is assumed to exist, the mass transfer rate is low enough ($\sim 5 \times 10^{-10} - 2 \times 10^{-8} M_{\odot}\text{yr}^{-1}$) that the accumulation ratio of the accreted matter $\alpha \sim 0$, and the WD mass hardly increases. The orbital period first decreases to ~ 0.8 d, then increases to $\lesssim 1.4$ d. When we include the effect of the CB disk with $\delta = 0.01$, the mass transfer rate maintains a relatively high value in the range of $\sim 3 \times 10^{-8} - 2 \times 10^{-7} M_{\odot}\text{yr}^{-1}$. A large fraction ($\sim 55\%$) of the transferred material is accreted by the WD, making the WD mass grow to $1.4M_{\odot}$ to trigger SNe Ia. Note that the binary orbit secularly shrinks until the orbital period P_{orb} reaches $\lesssim 0.4$ d. This evolutionary sequence may represent the formation history of systems like 1E 0035.4–7230 with narrow orbit, low-mass donor star and rapid mass transfer. Figure 2 shows the evolution of the donor and WD masses with time. It is clear that the donor star fills its Roche lobe when its age is ~ 2 Gyr for an initial donor star of $1.5M_{\odot}$, much longer than the ~ 20 Myr mass transfer time. This means that it is possible for the SD/Ch model to have a few Gyr delay time.

We have calculated the evolutions of a large number of WD binaries for a wide distribution of the initial input parameters. Figure 3 summarizes the final results of our binary evolution calculations for the distributions of the progenitor systems of SNe Ia in the $M_{\text{d,i}} - P_{\text{orb,i}}$ diagram. The bias and pane shading denote the distribution area of WD binaries with a

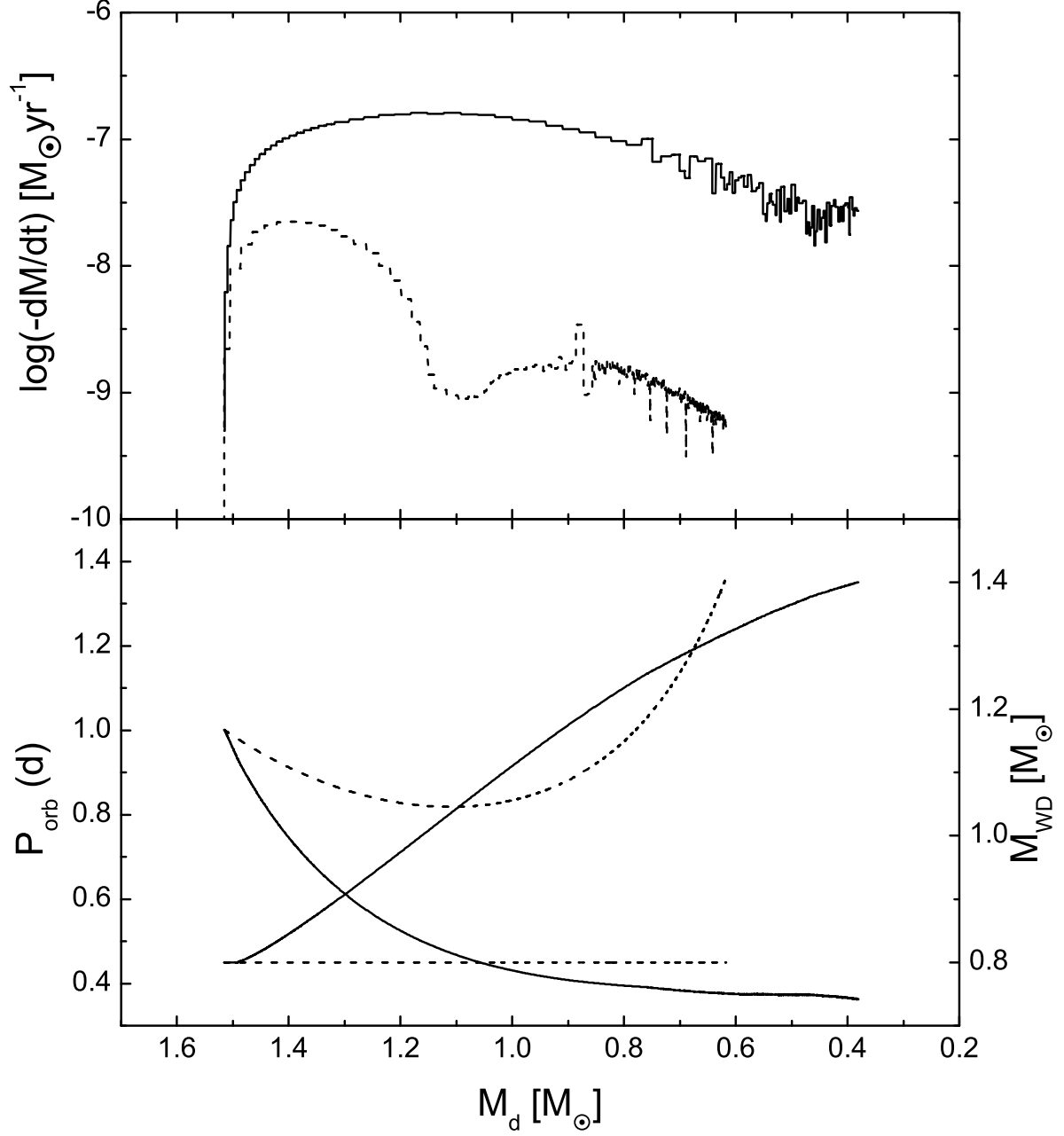


Fig. 1.— Evolution of the mass transfer rate \dot{M}_d , orbital period P_{orb} , and WD mass M_{WD} (lower curves) for a WD binary with $M_{\text{WD},i} = 0.8 M_\odot$, $M_{d,i} = 1.5 M_\odot$ and $P_{\text{orb},i} = 1\text{d}$. The solid and dashed curves denote the evolutionary tracks with $\delta = 0.01$ and 0 , respectively.

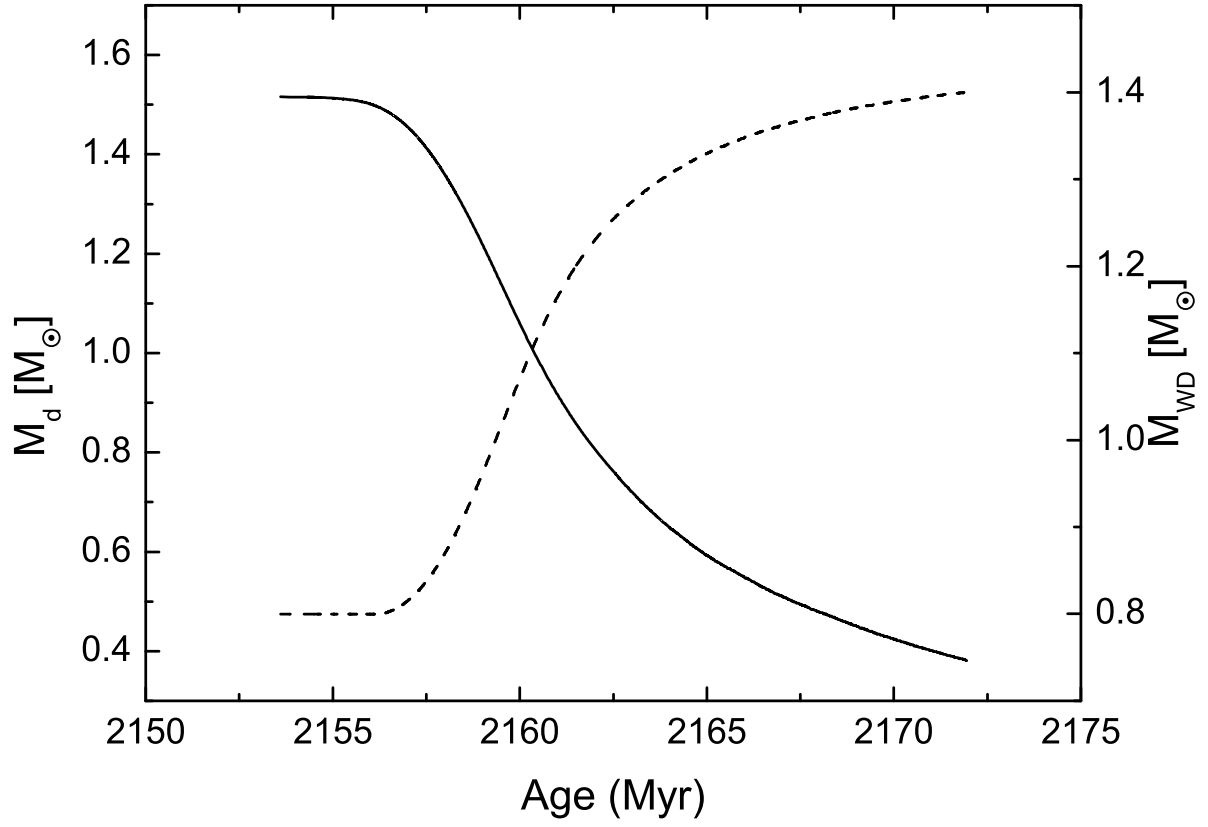


Fig. 2.— Evolution of the donor mass (solid curve) and WD mass (dashed curve) when $M_{WD,i} = 0.8M_\odot$, $M_{d,i} = 1.5M_\odot$, and $P_{orb,i} = 1d$.

WD of initial mass $M_{\text{WD},i} = 1.2$ and $0.8M_{\odot}$, respectively. Beyond these two areas, SNe Ia explosions cannot occur due to either low mass accumulation rate or unstable mass transfer. Compared with previous investigations, our CB disk model opens a possible evolutionary channel to SNe Ia for WD binaries with relatively low initial mass donor star of $\sim 1 - 2 M_{\odot}$ and hence long delay times.

To investigate the properties of a companion star that has survived the supernova explosion we have examined the consequences of heating and mass stripping by the impact of the supernova shell in a SN Ia on the secondaries. For the 24 ($M_{\text{WD},i} = 0.8M_{\odot}$) and 71 ($M_{\text{WD},i} = 1.2M_{\odot}$) WD progenitor binaries shown in Fig. 3, we calculate the mass loss fraction of the secondary star following the semianalytical method of Wheeler et al. (1975): the total ejected mass fraction is given by $\eta_{\text{ej}} = \eta_{\text{st}} + \lambda\eta_{\text{ev}}$, where η_{st} and η_{ev} are the stripped and evaporated mass fraction respectively, λ is an adjusted parameter we assume to include the uncertainties in estimating η_{ev} . Here we take the mass $M_{\text{SN}} = 1.4 M_{\odot}$ and the velocity $v_{\text{SN}} = 8500 \text{ km s}^{-1}$ for the SN ejecta (Marietta, Burrows & Fryxell 2000), and $\lambda = 0.5$. We find that the subgiant secondary (of mass $0.3 - 2.6M_{\odot}$) loses $\sim 9\% - 28\%$ of its mass after the SN. Generally the greater mass or binary separation, the smaller mass ejection fraction. Figure 4 shows the distribution of the secondary stars in the HR diagram just before and after the SN explosions. The latter could be compared with and testified by future observations of the companions of SNe Ia in young, nearby supernova remnants (e.g. Canal et al. 2001).

4. Discussion and Summary

Recently Mannucci, Della Valle & Panagia (2006), on the basis of observational arguments, suggest that there is a bimodal delay time distribution, in which about half of SNe Ia explode soon after their stellar birth, in a time $\sim 10^8$ yr, while the remaining half have a much wider distribution, well described by an exponential function with a decay time of about 3 Gyr. In the traditional SD/Ch model the rapid mass transfer required limits the donor stars to be more massive than $\sim 2M_{\odot}$ (Li & van den Heuvel 1997; Langer et al. 2000; Han & Podsiadlowski 2004). The innovation in this work is taking into account the effect of the CB disk on the mass transfer process. Our calculations indicate that the CB disk can significantly enhance the mass transfer rates, especially for systems with low-mass ($< 2 M_{\odot}$) donor stars, to trigger SNe Ia explosions. In particular, these progenitor systems are consistent with the time delay of $\sim 1 - 3$ Gyr between the formation of the progenitors and the SNe Ia. This scenario could also explain the existence of the peculiar SSS containing rapidly accreting white dwarf and very low-mass donor star like 1E 0035.4–7230 (e.g.

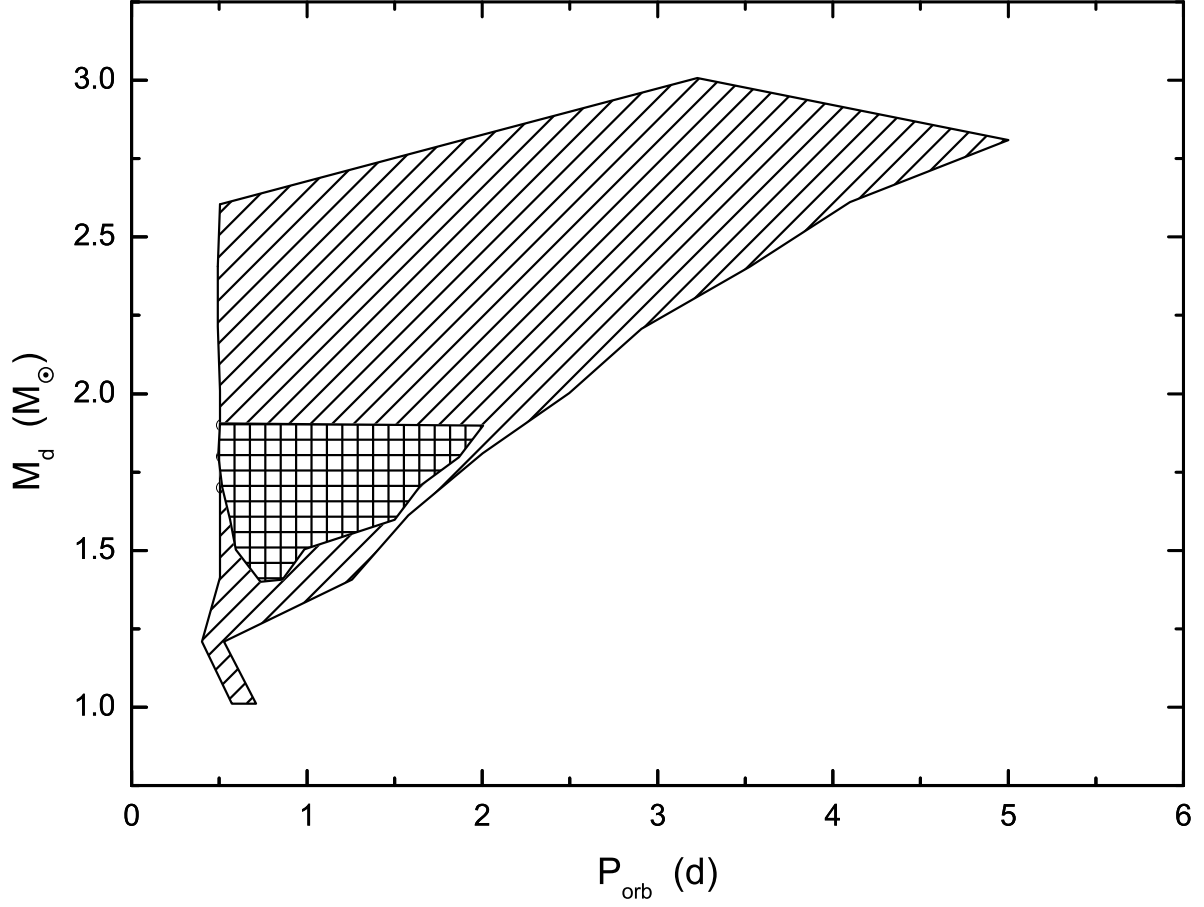


Fig. 3.— Distribution of the progenitor systems of SNe Ia in the $M_{d,i} - P_{\text{orb},i}$ diagram. The bias and pane shading denote the distribution area of WD binaries with a WD of initial mass $1.2M_\odot$ and $0.8M_\odot$, respectively.

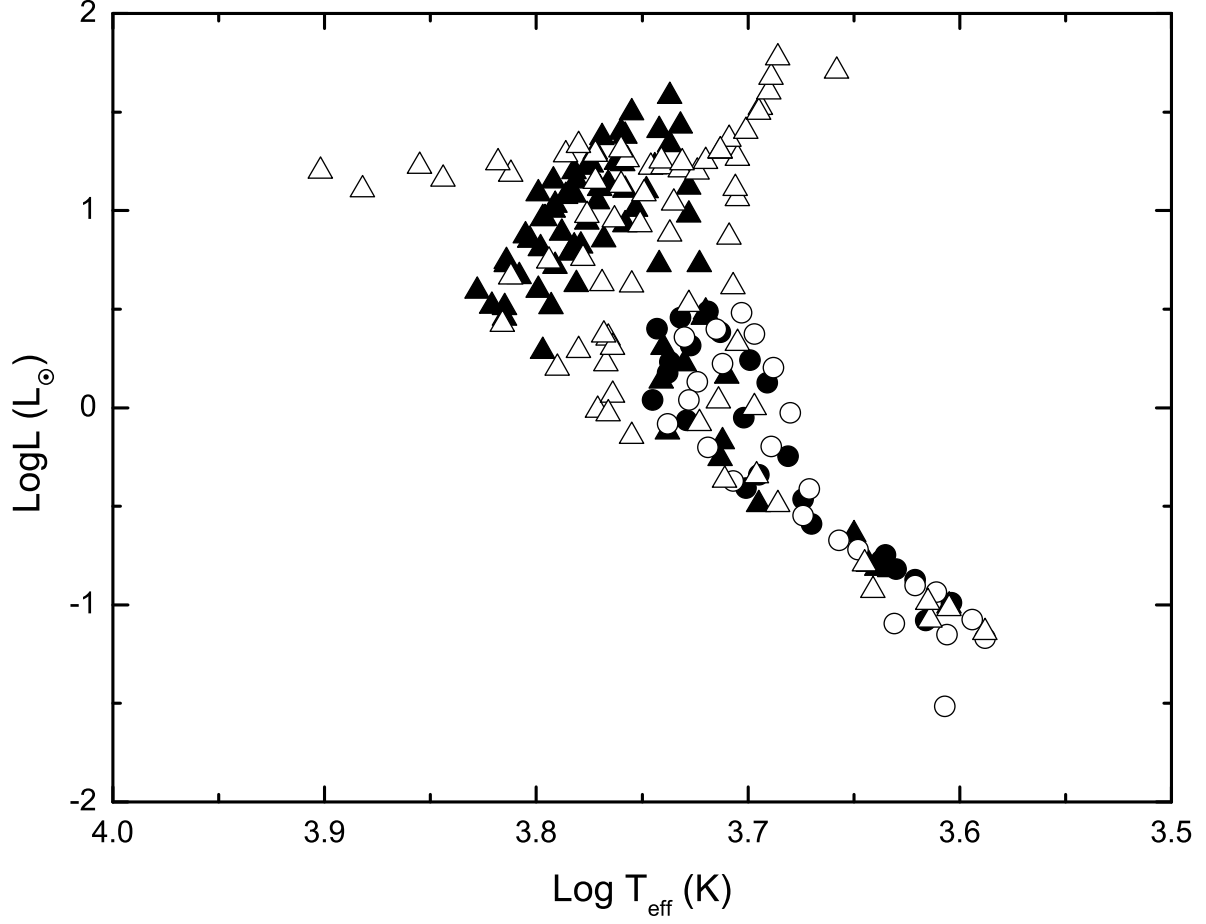


Fig. 4.— Distribution of the donor stars in the HR diagram when and after SNe Ia explosions occur. The triangles and circles denote the donor stars accompanied by an initial WD of $1.2M_{\odot}$ and $0.8M_{\odot}$, while the solid and open signs represent the cases of when and after SNe Ia explosions occur, respectively.

Taam & Spruit 2001).

Obviously there exist many uncertainties in the CB disk-driven mass accretion onto WDs in this work. First, the existence of the CB disk in supersoft X-ray binaries needs to be confirmed or disproved by future infrared observations (Dubus et al. 2004), as in GG Tau (Roddier et al. 1996). Second, the calculated mass transfer processes depend sensitively on the adopted value of δ , which is poorly known, and likely to change with time. This makes it difficult to estimate the contribution of the such binaries to SNe Ia. Third, we do not include the influence of rotation of the accreting WDs, which may cause the chandrasekhar mass to exceed $1.4M_{\odot}$, i.e. the super-Ch mass model (e.g. Uenishi et al. 2003; Yoon & Langer 2005; Howell et al. 2006). In conclusion, we want to emphasize that before judging which kind of progenitor scenario are more compatible with observations, we need to investigate carefully various mass transfer processes in WD binaries.

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